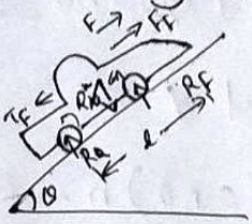
1. Derive the equation of motion and maximum tractive effort for a car inclined at an angle θ. Also give the expression of maximum gradability for a 4 wheel drive?



W = weight of the car

CG = Centre of gravity

b = wheel base

F = max forward acceleration

F_F = max tractive effort

R & F = Rear & front wheel

R_R & R_f = Total normal reaction at front & rear wheel

h = height from CG to road

IF = Inertia force $= m.f - W\sin\theta$

$= mf - \sin.\sin\theta$

$= W/g.f - W\sin\theta$ —①

$\Sigma V = 0$ —②

$\Sigma H = 0$ —③

Using —②

$W\cos\theta = R_f + R_R$ —④

Using ⑤

$$F_f = \frac{W}{g} \cdot F + W\sin\theta$$

$$MR_f = \frac{W}{g} \cdot F + W\sin\theta$$

$$R_f = \frac{W}{g} f + \frac{W}{M}\sin\theta \quad -⑦ \qquad \frac{W}{M}\left\{\frac{F}{g}+\sin\theta\right)$$

Taking moment about Y

$$R_f \times b + \left(\frac{W}{g} F\sin\theta\right) h = W\cos\theta \times l$$

$$\left(\frac{W}{g}\frac{\theta}{M} + \frac{W}{M}\sin\theta\right) b + \left(\frac{F}{g}+\sin\theta\right) h = \cos\theta l$$

$$\frac{b}{M}\left(\frac{F}{g}+\sin\theta\right) + \left(M\frac{F}{g}\sin\theta\right) h = \cos\theta l$$

$$\left(\frac{F}{g}+\sin\theta\right)\left(\frac{b}{M}+h\right) = \cos\theta l$$

$$\left(\frac{F}{g}+\sin\theta\right) = \frac{\cos\theta l}{\frac{b}{M}+h} \quad -⑧$$

$$\frac{F}{g} = \frac{\cos\theta l}{\left(\frac{b}{M}+h\right)} - \sin\theta$$

$$F = g\left[\left[\frac{\cos\theta l}{\frac{b}{M}+h}\right] - \sin\theta\right] \quad -⑨$$

$$R_f = \frac{W}{M}\frac{\cos\theta l}{\left(\frac{b}{M}+h\right)} \quad -⑩$$

$$= \frac{W\cos\theta l}{b + Mh}$$

$$R_R = W\cos\theta - R_f$$

$$= W\cos\theta\left[1 - \frac{l}{(b+Mh)}\right]$$

$$= W\cos\theta\left[1 - \frac{l}{b+Mh}\right]$$

$= W\cos\theta\left[\frac{b+\mu h-l}{b+\mu h}\right]$

$$F_f = \mu R_f = \mu\left(\frac{W\cos\theta l}{b+\mu h}\right)$$

Four wheel drive

$$F = R_f + F_f = \mu R_R + \mu R_F$$

$$\Sigma V = 0$$

$$W = R_f + R_R$$

$$\Sigma H = 0$$

$$(W/g) f = \mu R_R + \mu R_F$$

$$= \mu (R_R + R_f) = \mu W$$

$$(F/g) = \mu$$

$$\Sigma V = 0$$

$$W = R_R + R_f$$

$$\Sigma H = 0$$

$$(W/g) f = \mu R_R + \mu R_f$$

Assuming slip to occur at front wheels

for $R_f < R_R$ then

$$\Sigma M_R = 0 \quad 2\mu R_f = (W/g) F$$

$$R_F + (W/g) f_n = W$$