Assignment

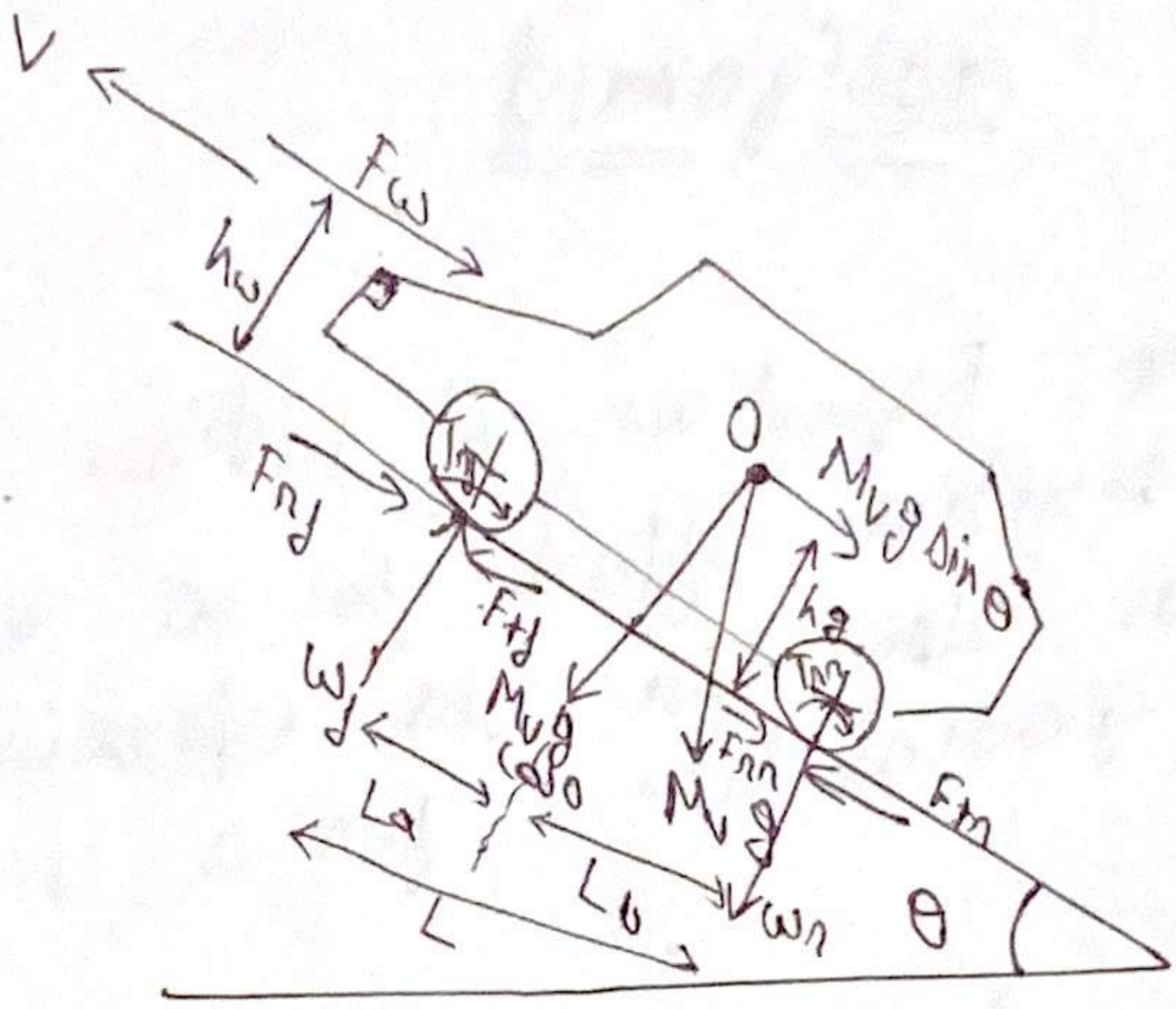
Q1:- Drive the Equation of motion and maximum tractive effort for a car inclined at angle θ. Also give the expression of maximum gradeability for a 4 wheel Drive.

Ans:- Equation of motion for a car inclined at angle θ.

→ Consider a vehicle of mass M, moving at velocity v, going up a slope of angle θ. The force propelling the vehicle forward, transmitted to the ground, through the drive wheels.

In a longitudinal direction, the major external forces acting on a two axle vehicle include:-

- The rolling resistance of the front and rear tires (F_{rf} and F_{rr}), which are represented by rolling resistance moment, T_{rf} and T_{rr}.
- The Aerodynamic drag / resistance. (F_w)
- Gravitation resistance / gradient resistance (F_θ)
- Acceleration resistance (F_a).



- Aerodynamic resistance (F_w)

→ It is due to presence of air and is given By :-

$$F_w = \frac{1}{2} \rho A_f C_D V^2$$

- Gravitation resistance (F_g)

→ It is due to inclination of slope at which car is moving.

Inclination angle is θ.

$$F_g = M_v g \sin\theta$$

where,

M_v is mass of vehicle

g is gravitational force.

- Rolling resistance :- (F_R)

$$F_R = F_{Rf} + F_{RN}$$

$$= W \cdot f_R \cos\theta$$

note :-
(f_R → dynamic friction
f_s → static friction
($f_R = N_R$) both same.

- Accelaration resistance (F_a)

$$F_a = ma$$

$$= M \frac{dV}{dt}$$

∴ Total driving resistance is equal to :-

→ sum of force due to rolling resistance (F_R), aerodynamic resistance / drag (F_w), gradient resistance (F_g) and accelaration resistance (F_a)

⇒ $F_{resistance} = F_R + F_w + F_g + F_a$.

*and, Traction force (F_T)

$$= F_T = F_{TJ} + F_{TR}$$

∴ According to Newton's Second law of motion dynamic equation of vehicle motion is given by :-

$$Ma = (\Sigma F_T) - F_{rf} + F_{rn} + F_w + F_g + F_a$$

$$\Rightarrow M\frac{dV}{dt} = (F_{tf} + F_{tn}) - (F_{rf} + F_{rn} + F_w + F_g + F_a)$$

$$\Rightarrow F_T = F_n + F_w + F_g + F_a$$

$$\Rightarrow \boxed{F_T = M_v g f_n \cos\theta + \frac{1}{2}\rho A_f C_D V_2 + M_v g \sin\theta + M\frac{dV}{dt}}$$

Now,

for Maximum tractive effort for a car inclined at angle θ.

→ We know that sum of moment about a point is zero. Here we have to consider the moments in front wheel and rear wheel. If we take the moment related to front wheel the distance corresponding to rear wheel should taken.

Considering front wheel :-

$$0 = -W_f L + M_v g L_b \cos\theta - (F_R \times r_{dyn}) - F_w h_w - M_v g h_g \sin\theta - M_v h_g \frac{dv}{dt}$$

*For a passenger cars, it is assumed that $h_w = h_g$. So above equation will be

$$\Rightarrow W_f = \frac{L_b}{L} M_v g \cos\theta - \frac{h_g}{L}\left(F_w + F_g + M_v g f_r \frac{r_d}{h_g}\cos\theta + M_v \frac{dv}{dt}\right)$$

Similarly for rear wheel;

$$\Rightarrow W_R = \frac{L_a}{L} M_v g \cos\theta + \frac{h_g}{L}\left(F_w + F_g + M_g f_r \frac{r_d}{h_g}\cos\theta + M_v \frac{dv}{dt}\right).$$

where, r_d, is effective radius of the wheel.

Now, Under balanced condition :-

- $M \frac{dv}{dt} = (F_{tf} + F_{tr}) - (F_{rf} + F_{rr} + F_w + F_g + F_a) = 0$
- $(F_t) - (F_R + F_w + F_g + F_a) = 0$
- $F_t - F_R = F_w + F_g + F_a$
- Also $F_R = M_v g f_r \cos\theta$

$$\Rightarrow W_f = \frac{L_b}{L} M_v g \cos\theta - \frac{h_g}{L}\left(F_t - F_{rn}\left(1 - \frac{a_g}{g}\right)\right)$$

$$\Rightarrow W_r = \frac{L_a}{L} M_v g \cos\theta - \frac{h_g}{L}\left(F_t - F_{rn}\left(1 - \frac{a_g}{g}\right)\right)$$

where $F_t = F_{tf} + F_{tr}$ is the total tractive effort

The first term on the right-hand side of above 2 equation is the static load on the front and rear axle when the vehicle is at rest on level ground.

i.e,

· For front wheel ± [illegible]

$$W_f = \frac{L_b}{L} M_v g \cos\theta$$

For rear wheel.

$$W_r = \frac{L_a}{L} M_v g \cos\theta$$

The second component is dynamic component of normal load.

Now,

The maximum tractive effort (F_{max}) that the tire - ground contact can support is described by the product of the normal load and the coefficient of road adhesion (μ). The adhesive capability between the tire and the ground is sometime the main limitation of vehicle performance.

Speed (Km/h)	μ of dry road	μ of wet road
50	0.85	0.65
90	0.8	0.6
130	0.75	0.55

for the front wheel drive vehicle F_{tmax} is

$$F_{tmax} = \mu W_f$$

for the rear wheel drive vehicle F_{tmax} is

$$F_{tmax} = \mu W_r$$

(where W_f and W_r is same as previous equation).

- Maximum Gradeability for 4 wheel drive :-

→ The maximum grade/slope that a vehicle will be able to overcome with the maximum force available from the propulsion unit is known as Gradability.

Assumptions :-

- The vehicle is moving very slowly ~ zero
- F_w and F_n is equal to zero
- Acceleration is zero
- Tractive force is maximum at nearly zero speed.

* Gradability at near stall conditions,

$$F_T = mg \sin\theta$$

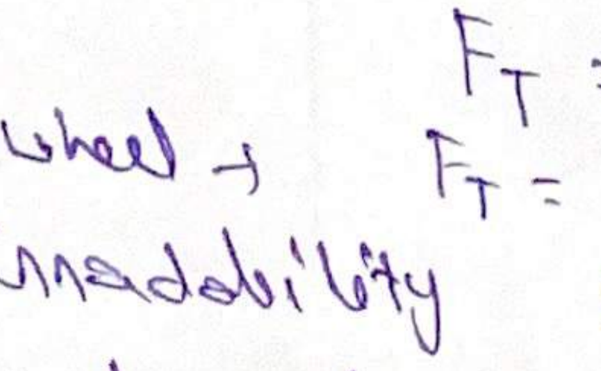
for 4 wheel → $F_T = F_{Td} + F_{TR}$

Gradability is calculated as ratio of rise/run on tangent of the slope angle = $\tan\theta$

∴ Maximum percentage grade = $100 \times \tan\theta$

$$= 100 \times \frac{F_T}{\sqrt{(F_g)^2 - (F_T)^2}}$$

$(mg)^2$



Q3:- Consider a car with following specification that is parked on a level ground. Find the load on the front and rear axel. $m = 1765\ kg$, $l = 2.84\ m$ $l_a = 1.22\ m$ $l_b = 1.62\ m$.

Ans:- Given:-

$m = 1765\ kg$

$l = 2.84\ m$

$l_a = 1.22\ m$

$l_b = 1.62\ m$

$g = 9.8\ m/s^2$

$\theta = 0°$ (∵ level ground).

for a car parked at level ground, the load on the front axel and rear axle is given by:-

For front wheel:-

$$W_f = \frac{L_b}{L} mg$$

$$W_f = \frac{1.62}{2.84} \times 1765 \times 9.8 = 9866.6\ N.$$

For rear wheel:-

$$W_r = \frac{L_a}{L} mg$$

$$W_r = \frac{1.22}{2.84} \times 1765 \times 9.8 = 7430.4\ N.$$

Q3:- What are the different parts of tires? Differentiate between types of tires on the basis of their construction.

→ The differents parts of the tyres are given below:

(i) Beads

→ Tire beads hold the tire to the rim, on the outer edge of the wheel. They are made of copper, brass, or brass-plated high tensile steel wires wound into a rubber band. Tire beads prevent the tyre from sliding out of the place when the wheels rolls.

(ii) Bead filler :-

→ Bead filler is a rubber compound inside the tire's beads. It provides stability to the lower sidewall and bead area. The density and stiffness of a tire's bead filler help to determine a tire performance characteristics.

(iii) Radial cord Body :-

→ The cord body gives the tyre strength and transmits cornering forces from the tread to the wheel. Rubber coated fabric cord, called body plies, make up the cord body. Body plies can be made of polyster, rayon nylon. Polyster is most commonly used.

(iv) Inner liner :-

→ The inner liner is a rubber compound bonded to the inside of the cord body that retains air under pressure. It functions like a inner tube.

(iv) Belt Plies :-

→ Belt plies are two or more strong layers of the cord just under the thread area of the tire. Its primary function is to provide strength and stability to the tire thread. Steel is most common material.

(v) Sidewall :-

→ The area of tire from the bead of the thread - the side of the tire is called the sidewall. It forms the protective covering for the cord body.

(vi) Thread :-

→ The tread is the portion of the tire that comes in contact with the road surface. The tread's compound and its design have to balance wear, traction, handling, fuel economy, resistance, and other characteristics of the tire. It design very greatly!

- Difference between types of tyre and their construction difference :-

→ On the basis of construction there are two types of tire :-

(i) Cross ply tyre :-

→ It is also known as bias ply tyre construction.

- This type has better a resistance against wear.
- They have good banding on road.
- It consist of carcass layers made from nylon cord which are placed diagonally along each other in the tread and the sidewalls, at angle of 50°.

- Multiple rubber plies stacked over each other which form a thick layer, which makes them less flexible as a result they are more sensitive to heating.

* Radial Ply Tyre :-

(i) It consists of a cross ply which is formed by textile cords running one bead to the other. each ply embedded at an angle of 90° to the rolling direction.

- They have lower rolling resistance.
- Longer life due to less heat generation.
- They provide better braking.
- Their soft sidewall are vulnerable.
- Make huge noise due to harder tread.